Developing a Broadband Circuit Model for the Cutler VLF Antenna

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Introduction

To investigate intermodulation problems related to two-frequency operation with the two identical elements comprising the Cutler VLF (15-30 kHz) antenna, a broadband (15-90 kHz) equivalent circuit was developed. The antenna, depicted in Fig.1 by a simplified wire model, consists of two 6-panel top-loading sections of ~1 km in radius

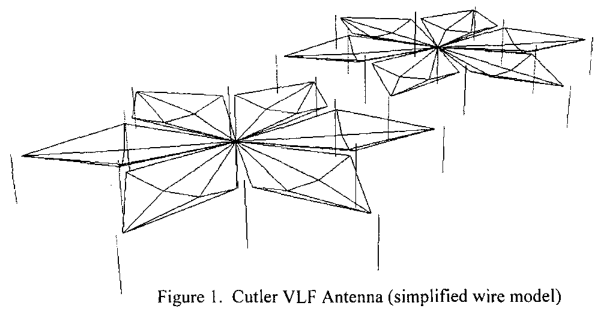

Figure 1. Cutler VLF Antenna (simplified wire model)

supported by 13 towers at an elevation of ~300 m. Two ~850-m coaxial transmission lines lead from a centrally located transmitter building to tuning systems located near the centers of each element. Although designed for use at a single frequency, current operational requirements make it important to consider increasing the information rate by operating the two antenna elements at two different frequencies. The purpose of this study was to provide an accurate two-port circuit model of the two antennas as part of a PSpice simulation of the entire transmitting system. Since the transmitters and the saturable reactors used for dynamic tuning involve substantial non-linearities, intermodulation products and harmonics at frequencies ranging up to the third harmonic were of concern. This dictated that an equivalent circuit was required that could accurately simulate terminal behavior over a frequency range of 15-90 kHz, i.e., the VLF range and its third harmonics.

Available Impedance Data

Although the Cutler antenna was built 40 years ago, only recently have broadband measurements been made of the input impedance. Because of the large size of the structure, test leads must be connected at awkward heights (~20m) and special precautions must be taken to avoid contact with the high voltages induced by the atmospheric effects. On the other hand, the electrical size of such an antenna relative to the wavelengths involved (10-20 km) is quite small. Since an electrically small antenna behaves essentially like a series RLC circuit [1], it has been possible for most purposes to characterize VLF antennas with only three parameters, the capacitance C_a, effective

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height h_e, and resonant frequency f_r. The antenna inductance L_a may then be determined from the resonant frequency. The resistance R_a is taken as the sum of the well-known radiation resistance [1] $R_r = 160\pi^2(h_e/\lambda)^2$ and a small loss component R_g mostly associated with the ground. Hansen's [2] characterization of Cutler may be summarized as follows (averaging values for North and South antennas): Static capacitance with unused antenna open = 120.5 nF, with unused antenna grounded = 122.7 nF, resonant frequency = 38.3 kHz, effective height = 148 m. The ~2 nF difference between the capacitance measured with the unused antenna open-circuited and grounded at its input is regarded as the mutual capacitance between the two antennas. Gish [3] improved the accuracy of this circuit by introducing a shunt capacitance $C_s \approx 14$ nF at the input to account for bushing and buswork capacitance. Although the series RLC model has served well in the design [1] of VLF antennas, and is somewhat improved by the addition of a shunt capacitance, it is still limited in application to operation at a single frequency and at frequencies less than f_r. These well-established concepts served as the starting point for developing a broadband equivalent circuit.

Only a very small amount of reliable measured data was available on the mutual impedance at Cutler. With some care, Watt and Smith [4] were able to measure the mutual impedance at two frequencies, giving values of $0.125 - j0.901$ ohms at 17.8 kHz, and $0.209 - j0.698$ ohms at 24.0 kHz.

Using a special-purpose network analyzer, Gish and Hopkins of Pacific-Sierra Research (PSR) recently measured the input impedance at Cutler over a very wide frequency band [5]. The data obtained was felt to be accurate in magnitude but known to include some scatter in phase. Since the impedance is largely reactive, this meant that the resistive part was frequently in question. However, by comparing their data with theory, it was possible to remove most of the scatter in the resistance. Consequently, in what follows this smoothed data is regarded as "measured" and identified by the subscript "psr."

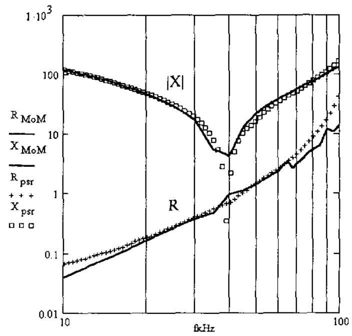

Figure 2. Cutler impedance as measured, R_{psr}, X_{psr}, and from theory, R_{MoM}, X_{MoM}.

Using the approximate wire model of Cutler shown in Fig. 1, with conductors increased in diameter to compensate for reduction in number, etc., the method of moments (MoM) [6] was used to determine theoretical values for even- and odd-mode terminal impedances and, from this, the self and mutual impedance over the frequency range of interest. In this analysis ohmic losses, which are known to be small, were not included. Thus it was necessary to utilize both the theoretical and the measured impedance responses in the design of an equivalent circuit.

Wave Impedance Models

In analyzing spherical antennas, Chu [7] derived an equivalent circuit for the TM and TE mode spherical mode wave impedances. In the radiation field of an electrically short antenna, the TM_{10} mode is dominant. Chu's equivalent circuit for the TM_{10} mode wave, i.e., the ratio of E_θ to H_ϕ in an outgoing wave, is shown in Fig. 3, the circuit elements being defined in terms of the radius r measured from the phase center of the radiated wave. In the case at hand, this radius was set equal to Wheeler's *effective radius* [7], r_e, the radius of a circular disk that, elevated at the effective height h_e, would yield the observed antenna capacitance C_a, neglecting electric field fringing. This is consistent with the relation $C_a = \varepsilon_0 \pi r_e^2 / h_e$. For Cutler, with $C_a = 110.6\,\text{nF}$ and $h_e = 139.2\,\text{m}$, it follows that $r_e = 744\,\text{m}$, yielding $C_1 = \varepsilon_0 r_e = 6.59\,\text{nF}$, $L_1 = \mu_0 r_e = 935\,\mu\text{H}$, and $R_1 = \sqrt{\mu_0/\varepsilon_0} = 377\,\Omega$. The asymptotic resistance of this circuit at low frequencies varies as the square of the frequency, making it behave, in this respect, similarly to the radiation resistance of the antenna. What is not so clear is how to couple a second-order high-pass circuit of this type to two single-antenna circuits to provide for mutual coupling and radiation loading. Many alternatives to this circuit were explored before satisfactory results were obtained.

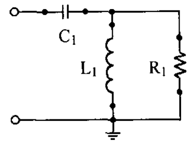

Figure 3. TM_{10}-mode equivalent circuit.

Results

After a investigating many circuit topologies, using Mathcad and monitoring rms values of the self and mutual impedance departures from target (measured and theoretical) data, the circuit shown in Fig. 4 was obtained. In this circuit, the reactive elements of the two pi networks were initially set to the values noted above [2,3]. The mutual impedance coupling the vertical currents in the downleads was modeled by two transformers coupled with a 377-ohm lumped-element model of a transmission line to simulate the phase shift with frequency. A parallel RL circuit was inserted in each pi circuit to simulate a small frequency-dependent ground system losses. In addition, a 377-ohm transmission line with a 2nd-order filter with a cutoff frequency of 130 kHz couples

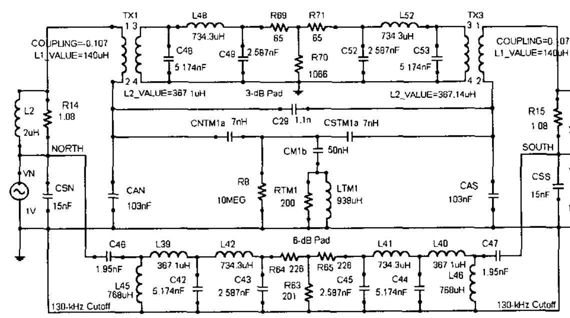

Figure 4. Cutler modeled as two coupled pi circuits.

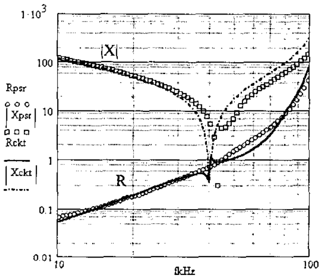

Figure 5. Impedance, Rckt, Xckt, from the circuit of Fig. 4, and as measured by PSR [5], Rpsr, Xpsr.

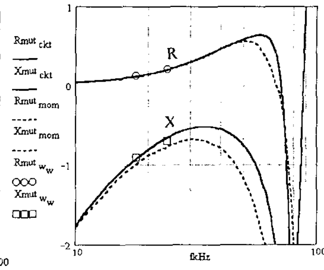

Figure 6. Mutual impedance from circuit of Fig. 4, $Rmut_{Ckt}$, $Xmut_{Ckt}$; MoM, $Rmut_{MoM}$, $Xmut_{MoM}$; and measured [4], $Rmut_W$, $Xmut_W$.

the input voltages to simulate high-frequency coupling and radiation. Finally, a high-pass radiation load, simulating TM_{10} loading, coupled the top loading capacitances. These loads were modified by separating the input capacitance C_l in Fig. 3 into capacitances, C_{la} and C_{lb}, with C_{lb} placed in the common ground leg, and by varying the ratio C_{la}/C_l for a best fit to the measured impedance. In addition, the resistance terminating these loads was varied. In Fig. 5 the input impedance computed from the circuit in Fig. 4 is compared with that measured by Gish and Hopkins [5]. More importantly, perhaps, is the comparison between the mutual impedance obtained from the equivalent circuit and that from theory and measurement shown in Fig. 6. The accuracy of the equivalent circuit, although imperfect, was felt to be adequate for the subsequent analysis. Future work will involve using a distributed model of each top load as a radial transmission line to try to further improve the accuracy of the equivalent circuit.

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